Probability & Bridge

NKy Summer Getaway Sectional
August 12, 2017
Goals

• Practical bridge advice
• Improve how we think at the table
• Get better results in tough contracts

• **NOT**: combinatorial mathematics or statistical equations.
Simple Chances

- Flip a coin
- Roll a die
- Take a finesse
Flip a Coin

• Coin has two sides (2 Total cases)
• One side is up (1 Specific Case): Heads or Tails
• *a priori* probability = \( \frac{\text{specific}}{\text{TOTAL}} = \frac{1}{2} = 50\% \)

• Each coin toss is INDEPENDENT of the prior event (Coins have no memories)

• **Probability of success for both of 2 independent events is the product of the probability of each:**
  - Two coins giving heads (HH): \( \frac{1}{2} \times \frac{1}{2} = 25\% \)
  - 3 Coins giving Heads (HHH): \( 25\% \times \frac{1}{2} = 12.5\% \) etc...
Coin Quiz

- Which sequence of 10 coin tosses is more likely?

Sequence A: HHHHHHHHHHH 0.0977%

Sequence B: THTTHTTHHHTT 0.0977%
Rolling a Die

• A standard die has 6 sides – 6 Total cases
• One side shows up – 1 Specific Case.
• The roll of any one die each number has an equal probability of $1/6 = 16.67\%$
• Each role is INDEPENDENT (die has no memory)

• Q: With two fair dice, what is the probability of rolling a 7?
Rolling a 7

- Outcome table (6x6=36 Total Cases)

- Frequency Table:

<table>
<thead>
<tr>
<th>#</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Tot</th>
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<tr>
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<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>%</td>
<td>3.84</td>
<td>7.69</td>
<td>11.54</td>
<td>15.38</td>
<td>19.23</td>
<td>26.09</td>
<td>19.23</td>
<td>15.38</td>
<td>11.54</td>
<td>7.69</td>
<td>3.84</td>
<td>100</td>
</tr>
</tbody>
</table>

Rolling a 7 is 26.09%

Craps (2 or 12) is 7.69%, the SUM of 2% and 12% (3.84 + 3.84).

For independent events, A and B is the product $P_A \times P_B$, while A or B is the sum $P_A + P_B$.
When is a finesse like a coin flip?

• When we lack INFORMATION!!! \((a\ priori)\)
• 2 Cases: Win or lose finesse
• Just like coin: Heads or Tails
• Therefore Finesse is 50%, \textit{lacking other information}
Bridge Hands – BIG NUMBERS

• 635,013,559,600 - # of ways to deal 13 cards.
• 53,644,737,765,488,792,839,237,440,000 - the number of possible ways to deal all 52 cards, 13 at a time.
• Odds of 4 players being dealt all 13 cards in one suit:
  1 in 2,235,197,406,895,366,368,301,559,999
Which Hand is More Likely?

♠ AKQJ1098765432  ♠ AK32
♥ K984  ♦ Q10
♦ J107  ♣
<table>
<thead>
<tr>
<th>What I gave you:</th>
<th>What you saw:</th>
<th>What you assumed:</th>
</tr>
</thead>
<tbody>
<tr>
<td>♠ AK32</td>
<td>♠ AKxx</td>
<td>♠</td>
</tr>
<tr>
<td>♥ K984</td>
<td>♥ Kxxx</td>
<td>♥</td>
</tr>
<tr>
<td>♦ Q10</td>
<td>♦ Q10</td>
<td>♦</td>
</tr>
<tr>
<td>♣ J107</td>
<td>♣ J10x</td>
<td>♣</td>
</tr>
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</table>
**SUIT SPLITS**

- # of specific cases / # Total Cases (approximately)
- # Total Cases = $2^m$ (where m=# missing cards)
- Study 2 – 7 missing cards (4 – 128 Tot. Cases)

<table>
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<tr>
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<td>7</td>
<td>0.5</td>
<td>7</td>
<td>30.5</td>
<td>62</td>
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<td>6</td>
<td>1.5</td>
<td>14.5</td>
<td>48</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>28</td>
<td>68</td>
<td></td>
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<tr>
<td>4</td>
<td>10</td>
<td>50</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>52</td>
<td></td>
<td></td>
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</table>
## DROP Missing Honors

<table>
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<tr>
<th>Honor Drop Missing:</th>
<th>%</th>
<th>H</th>
<th>Hx</th>
<th>Hxx</th>
<th>TOT</th>
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<tbody>
<tr>
<td></td>
<td>8</td>
<td>0.4</td>
<td>4</td>
<td>18</td>
<td>22.4</td>
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<tr>
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<td>7</td>
<td>1</td>
<td>9</td>
<td>27</td>
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<td></td>
<td>6</td>
<td>2.4</td>
<td>16</td>
<td>36</td>
<td>54.4</td>
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<td>27</td>
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<tr>
<td></td>
<td>2</td>
<td>52</td>
<td>48</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Suit Combinations

• How to play suits – wrong when done alone – right when done in the context of whole hand.
• Know # tricks needed.
• Vacant Spaces 13 each, reduced by information.
**a priori** Suit Combinations

1. **K3** opposite **6710QA** Do you finesse for the 10? Why/Not?

2. **AJ975 -- K1086** - You play the K. LHO plays the 2 RHO the 3. You play the 6 to dummy. LHO plays the 4. **Finesse or drop?**

3. **AQ97** opposite **K108** - You play the 10 to the Q and the 7 to the K, RHO playing 2,4. LHO playing 3, 5. Now you continue the 8 and LHO plays the 6. **Finesse or drop?**

4. **AJ1074** opposite **52** (need 3 tricks)

5. **AKQ74** opposite **52** (need 4 tricks; Need 5 tricks) **NO SIDE ENTRIES.**
Suit Combination 1

• **K3** opposite **6710QA** Do you finesse for the 10? Why/Not?
  
  – Absent information the finesse is worth 50%.
  – If we are looking for the J, then we can win when the J is singleton, doubleton or Jxx in either hand.
  – Combining those chances results in $2.4 + 16 + 36 = 54.4\%$ so cashing tops is better.

What Information would make you change your play?

- Count of the hand → split known
- # Tricks needed from this suit
- Avoid having DANGER HAND on lead
- Can ruff out the suit
Suit Combination 2

- **AJ975 -- 6810K** - You play the K. LHO plays the 2. RHO the 3. You play the 6 to dummy. LHO plays the 4. **Finesse or drop?**
  - Any 2-2 break is 40% while any 3-1 break is 50%.
  - 2-2 has 6 cases. 3-1 has 8. So the specific case for 3-1 is less likely (absent additional information).
  - The Qxx w/ LHO is 6.21%. The Qx with RHO is 6.78%.
  - The ratio 6.78/13 = 52.2%.
  - Vacant spaces says LHO has 11 while RHO has 12 before declarer’s choice. 12/23 = 52.2% the Q is with RHO.

What Information would make you change your play?
Suit Combination 3

• **AQ97** opposite **810K** - You play the 10 to the Q and the 7 to the K, RHO playing 2,4. LHO playing 3, 5. Now you continue the 8 and LHO plays the 6. **Finesse or drop?**
  
  – You have seen 3 insignificant cards from LHO and 2 from RHO.
  
  – That leaves 10 spaces for LHO and 11 for RHO.

  – Therefore the probability that the J is with LHO is $11/(10+11) = 52.4\%$. **DROP**

What Information would make you change your play?
Suit Combination 4

- **AJ1074** opposite **52** – **Goal: 3 tricks**
  - We are missing the **KQ9863**
  - From the chart, 3-3 happens 36% of the time and 4-2 happens 48% of the time.
  - Missing 6 cards there are $2^6 = 64$ total cases.
  - 6 cards taken 3 at a time counts to 20
  - 6 cards take 2 (or four) at a time counts to 30

- Any 3-3 means we win 3 tricks. Any 1-5 or 0-6 and we fail. Ignore these.

- 4-2/2-4 is where we can gain advantage. A finesse helps only when KQ63 – 89 is the distribution (One Case), and Hx-Hxxx (4 Cases).

- Playing small to the A and SMALL from length works for the 8 cases where Hxxx – Hx / Hx – Hxxx plus the 2 cases KQ-9863 or 9863-KQ. 10 Cases

- Ace then small is better *a priori* than the 1st round finesse.

What Information would make you change your play?
Suit Combination 5

- **AKQ74** opposite **52** (need 4 tricks; Need 5 tricks)
- Needing 5 tricks, we play top down, for a 36% chance (3-3 split).
- Needing 4 tricks we can do better. If we duck the first trick we will get 4 tricks if the suit splits 3-3 (36%) or 4-2/2-4 (48%). This improves our chances to 84%. Much better than playing the suit top down (remember we have no outside entry).

What Information would make you change your play?
## Suit Split Probability

Richard Pavlicek  
Bridge Site:  
[http://www.rpbridge.net/](http://www.rpbridge.net/)

Case: Missing 6 cards including the Q

Best way to answer the question “WHY??”

Now the fun starts!

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>West</th>
<th>East</th>
<th>Ways</th>
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<td></td>
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<td>Qxxxxxx</td>
<td>—</td>
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<td>24</td>
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<tr>
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<td>✔</td>
<td></td>
<td></td>
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<td>x</td>
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<td>3</td>
<td></td>
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<tr>
<td>11</td>
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<td>✔</td>
<td>x</td>
<td>Qxxxx</td>
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<td>✔</td>
<td>—</td>
<td>Qxxxxx</td>
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<td>24</td>
<td>0.75</td>
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### Goal to win

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<th>B</th>
<th>C</th>
<th>D</th>
<th>Totals</th>
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<td>Q, Qx, Qxx onside</td>
<td>3</td>
<td>16</td>
<td>871</td>
<td>27.05</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Ruff the 4th Club</td>
<td>3</td>
<td>16</td>
<td>739</td>
<td>22.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Play off 3 rounds of Clubs</td>
<td>6</td>
<td>32</td>
<td>1742</td>
<td>54.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Q drops in 2 rounds</td>
<td>4</td>
<td>12</td>
<td>598</td>
<td>18.57</td>
<td></td>
<td></td>
<td></td>
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</table>
Combining Chances

• ♠AJ1097 opposite 543

• What is the probability you can score 4 tricks?
  – p Both ♠ A&K are onside: 24%
  – p Honors are split 52%
  – p Both honors are on your right 24%

P Success = 24% + 52% = 76%

• We Need: Finesse in Suit A, and if that fails a 3-3 break in suit B. What is the probability we make our contract?
  – 50% Finesse wins + 50% Finesse loses X (36% 3-3 split) = 68%

  \[ P_{1\text{-success}} + P_{1\text{-fail}} \times (P_{2\text{-Success}}) \]

  – Notice that independent plays (NOT finesses) chances ADD.
  – Requiring 2 or more finesses multiplies fractions.
Analyze 1st – Plan 2nd

The Whole Bridge Hand
• Use ALL your information – Bidding & Play
• Start with a flexible picture of declarer/opponent
• Count hand winners and losers ("off the top") and SLOW LOSERS
• Count entries
• Count stoppers in threat suits.
• Count HCP - Your total and their total
• Combine your chances – Source of Tricks/Trick Packets
• Avoid the DANGER HAND. Assume perfect defense.
• Modify plan as you learn - Show-outs are GOLD

Use All Information
Common Inferences
• Opening bids show 12 HCP + and 5+ Cards in a Major.
• 1 NT is typically 15-17.
• Weak 2 for 7-8 HCP and 6 cards
• a 3-bid less (~6) and 6-7 cards

An INERENCE is what we judge

INFORMATION is what we see and know. (Show outs are INFORMATION)
Tips

Combining Chances (Mutually Exclusive events)
• Plan for failure – Stay ALIVE.
• Find chances that create options
• Cash winners in your long side suit (drop honors)
• Finesse long suits into safe hands when necessary.
• Avoid finesses completely if possible
• Leave short suits (no extra chances) until the end.

Steve’s Tips:
• Always choose the plan with the best probability
• Find a good plan? **LOOK AGAIN. FIND A BETTER ONE**
• When faced with equal choices, choose the option that allows you to **STAY ALIVE** longest (Take more chances)
• **Any Chance** is better than NO Chance
• **NEVER** take a **PRACTICE FINESSE**.
A Simple Hand?

Contract: 4♠, Opponents pass throughout

N: ♠ QJ432
W: ♥ AK2
E: ♦ Q3
S: ♣ J456

N: ♠ 1098
W: ♥ 1064
E: ♦ AK65
S: ♣ KQ2

Lead ♥3

Analysis:
Winners: 5  Losers: 3 Fast, 1 Slow
Entries: W3 and E2
Stoppers: ♥2,
Source of Tricks: ♠(3), ♠(2)
Improving Your Plan

Contract 6♥, no opposing bidding

♠ AQ  ♥ AKJ72  ♦ AQ  ♣ J456
♠ 98  ♥ Q10654  ♦ J65  ♣ AK10

ANAYLSIS:
Winners: 9  Losers:  0 Fast, 3 Slow
Entries:  W5 and E3
Stoppers:  1♠  1♦  2♣
Source of Tricks:  ♠ (3);  ♦(2)
Plan 6♥
Improving your plan 1

A Novice (or finesse-aholic)

- sees 3 finesses, draw trumps in 2-4 rounds and begin.
- 3 Finesses here are independent (different suits, different players) so the odds of all 3 are $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ or 12.5%.
- They need only 2 of the 3 finesses. How do you calculate the probability? Think: 2 winning finesses is the same case as one losing finesse or 50%.
Improving your plan 2

Intermediate Player:

• After pulling trumps if the ♠ finesse works and they split 3-3, they can pitch a losing ♠ on the long ♠,

Combining chances that way means:

– 50% ♠ finesse x 36% 3-3- split = 18%
– 50% ♥ Finesss x 82% remaining = 41% or
– TOTAL CHANCE: 59%

a useful improvement.
Improving your plan 3

Expert Player:

- Cash the ♠ AK = 18% +
- % Time North has 0, 1, 2 ♠ = 26% (NOT Q, Qx)
- ♠ Finesse (50% x 56%) = 28%
- TOTAL 72+% 

• Look deeper! Treat the hand as “one of 2 finesses”: Finesse ♠. If win, cash ♠ tops. If no ♠Q, finesse ♠. If lose, long ♠ goes away on 3rd ♦. Likewise if ♠ finesse loses, we need only the ♦ finesse to win (Pitch the losing ♠ on the ♠) - a 75% play.
Bonus Problem

6♠, No opposition bids. What is the likelihood of success?
Trumps split 2-1

ANSWER: Take NO Finesses. Draw 2 rounds of Trump and 2 rounds of ♥. Then play off ♦AKJ in that order. No matter who wins they have to either give you a free ♠finesse or a ruff sluff – either way we lose only 1 ♦ trick.

PROBABILITY OF SUCCESS: 100%
Either N or S must lead and give us the rest of the tricks. You play like an expert!

**ANSWER:** Take **NO** Finesse. Draw 3 rounds of Trump and 2 rounds of ♥. Then play off ♦AKJ in that order. No matter who wins they have to either give you a free ♠ finesse or a ruff sluff – either way we lose only 1 ♦ trick.

**PROBABILITY OF SUCCESS:** 100%
Other Uses for “p”

• When to bid Game, Small Slam, and Grand Slam
  – The likelihood of success must match or exceed breakeven
### Game, Slam & Grand Odds

<table>
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<th>Type</th>
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<th>Grand Slam</th>
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<td></td>
<td>V</td>
<td>NV</td>
<td>V</td>
</tr>
<tr>
<td>Score+</td>
<td>620</td>
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<tr>
<td>Score-</td>
<td>-100</td>
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<tr>
<td>Not Bid</td>
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<tr>
<td>Win</td>
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<tr>
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<tr>
<td>IMPs W</td>
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<td>13</td>
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<tr>
<td>IMPs L</td>
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<td>-5</td>
<td>-13</td>
</tr>
<tr>
<td>Break Even</td>
<td>38%</td>
<td>45%</td>
<td>50%</td>
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</table>

*If opponents bid game, then bidding a Grand Slam is Poor.* With 12 tricks, a slam gains +11NV & +13V IMPS, the grand loses -11NV Imps and -13V, swinging -22NV & -26V Imps. Avoid grand slams when they only bid game. Need 14 tricks.
Useful *a priori* %:

- Have FUN playing BRIDGE 100%
- Need 1 of 2 finesses (same hand) 76%
- Missing cards split 3-2 68%
- Missing cards split 4-3 62%
- Q drops in 3 rounds when holding 7 cards 54.4%
- Pure finesse 50%
- Need 2 finesses of 3 available 50%
- Q drops in 3 rounds when holding 6 cards 37%
- Suit splits 3-3 36%
- Need 3 finesses of 4 available 31%
- Need 2 finesse of 2 available 25%
- Need finesse & 3-3 split 18%
- Need 3 finesses 12.5%
References

THANK YOU FOR YOUR KIND ATTENTION!

See you at the tables!

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